Name:

SM275 · Mathematical Methods for Economics — Fall 2019 · Uhan

Exam 2 – 28 October 2019

Instructions

- You have until the end of the class period to complete this exam.
- You may not use your calculator.
- You may not consult any other outside materials (e.g. notes, textbooks, homework, computer).
- **Show all your work.** To receive full credit, your solution must be completely correct, sufficiently justified, and easy to follow.
- Keep this booklet intact.

Problem	Weight	Score
1	1	
2	1	
3	1	
4	1	
5	1	
6	1	
7	1	
8	2	
9	1	
10	2	
11	1	
12	1	
13	1	
14	1	
15	2	
16	2	
Total		/ 200

For the problems on this page, let

$$A = \begin{bmatrix} 2 & 0 \\ -1 & 1 \\ 3 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 4 & -2 \\ 0 & 3 \end{bmatrix} \quad C = \begin{bmatrix} -1 & 4 \\ 3 & 5 \\ -2 & 0 \end{bmatrix}$$

If the quantity you are asked to compute is undefined, briefly explain why.

Problem 1. Compute $A - 2C$.

Problem 2. Compute AB.

Problem 3. Compute AC.

Problem 4. Compute B^{-1}.

For the problems on this page, let

$$A = \begin{bmatrix} 2 & 0 \\ -1 & 1 \\ 3 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

If the quantity you are asked to compute is undefined, briefly explain why.

Problem 5. Compute A^T.

Problem 6. Compute AB.

Problem 7. Compute BA^TC. What size is BA^TC?

Consider the system of linear equations below.

$$\begin{aligned}
2x_1 + \ 4x_2 - 2x_3 + 2x_4 + 4x_5 &= 2 \\
x_1 + \ 2x_2 - \ x_3 + 2x_4 \qquad\quad &= 4 \\
3x_1 + \ 6x_2 - 2x_3 + \ x_4 + 9x_5 &= 1 \\
5x_1 + 10x_2 - 4x_3 + 5x_4 + 9x_5 &= 9
\end{aligned}$$

The reduced row echelon form of the augmented matrix for this system is

$$\begin{bmatrix} 1 & 2 & 0 & 0 & 3 & 2 \\ 0 & 0 & 1 & 0 & -1 & 4 \\ 0 & 0 & 0 & 1 & -2 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Problem 8. What are the solutions of this system? Write the solutions in vector form. If there are no solutions, simply state so.

Problem 9. How many solutions does this system have?

For this page, let $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 4 & 0 \end{bmatrix}$.

Problem 10. Compute A^{-1}. (You may assume it exists.)

Problem 11. Does $|A| = 0$? Briefly explain without computing $|A|$.

For this page, let

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 4 & 1 & 3 \\ 3 & 0 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & -7 & 2 \\ 1 & ?? & 2 \\ 1 & 4 & 2 \end{bmatrix} \qquad C = \begin{bmatrix} 4 & ?? & 5 \\ 0 & -1 & ?? \\ 0 & 0 & 2 \end{bmatrix}$$

Note that some of the entries in the above matrices are deliberately missing.

Problem 12. Compute $|A|$.

Problem 13. Compute $|B|$.

Problem 14. Compute $|C|$.

Problem 15. Recall the national income model

$$Y = C + I_0 + G_0 \tag{1}$$
$$C = a + bY \qquad (0 < b < 1) \tag{2}$$

where

$$\begin{aligned} Y &= \text{national income} \\ C &= \text{consumer expenditure} \\ I_0 &= \text{business expenditure (i.e., investment)} \\ G_0 &= \text{government expenditure} \end{aligned}$$

Suppose $I_0 = 7$, $G_0 = 2$, $a = 3$, $b = \frac{1}{3}$. Use Cramer's rule to find the national income Y and consumer expenditure C.

Problem 16. Consider an economy with two industries. Industry 1 manufactures product 1, and industry 2 manufactures product 2.

Industry 1 uses 0.4 dollars of product 1 and 0.5 dollars of product 2 for every dollar of product 1 it manufactures. Industry 2 uses 0.1 dollars of product 1 and 0.3 dollars of product 2 for every dollar of product 2 it manufactures.

Consumers demand $20,000 of product 1 and $10,000 of product 2.

Let

$$
\begin{aligned}
x_1 &= \text{output of industry 1, in dollars} \\
x_2 &= \text{output of industry 2, in dollars}
\end{aligned}
$$

Write the Leontief input-output matrix equation for this model — i.e., the matrix equation that ensures that each industry's output is equal to the input demand and the final demand for its product. Your answer should look like this:

$$
\begin{bmatrix} \text{some matrix} \\ \text{with numbers} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \text{some other matrix} \\ \text{with numbers} \end{bmatrix}
$$